AMERICAN UNIVERSITY OF BEIRUT
ELECRICAL & COMPUTER ENGINEERING DEPARTMENT

EECE 460	**Control Systems** **Quiz I-Solution**	**Fall 2015**

Problem 1 (6 pts)

Find the Laplace transform of the following function

$$g(t) = \begin{cases} 0 & t \le 0 \\ 2 & 0 < t < 2 \\ t & 2 \le t < 3 \\ e^{t-3} + 3 & t \ge 3 \end{cases}$$

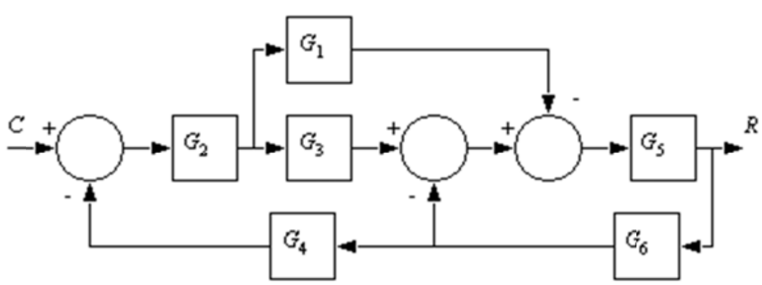
Answer

$$g(t) = 2[u(t) - u(t-2)] + t[u(t-2) - u(t-3)] + \left(e^{t-3} + 3\right)u(t-3)$$
$$g(t) = 2u(t) - 2u(t-2) + tu(t-2) - tu(t-3)] + 3u(t-3) + \left(e^{t-3}\right)u(t-3)$$
$$g(t) = 2u(t) + (t-2)u(t-2) - (t-3)u(t-3) + \left(e^{t-3}\right)u(t-3)$$

$$G(s) = \frac{2}{s} + \left(\frac{e^{-2s}}{s^2}\right) - \left(\frac{e^{-3s}}{s^2}\right) + \left(\frac{e^{-3s}}{s-1}\right)$$

Problem 2 (7 pts)

The block diagram of an LTI system, with input C(s) and output R(s), is shown below

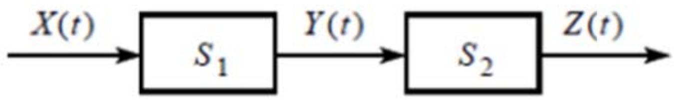


Determine the system transfer function

$$TF = \frac{R(s)}{C(s)} = \frac{G_2G_3G_5 - G_2G_1G_5}{1 + G_2G_3G_5G_6G_4 + G_5G_6 - G_1G_2G_4G_5G_6}$$

Grading system: - 1 for any incorrect term

Problem 3 (7 pts)
Two linear time-invariant systems connected in series as shown below



The differential equations given below represent the input-output relationships for each system

$$\frac{d^2y(t)}{dt^2} + y(t) = x(t) \text{ and } \frac{dz(t)}{dt} - 6z(t) = \frac{dy(t)}{dt}$$

Find the transfer function of the system.

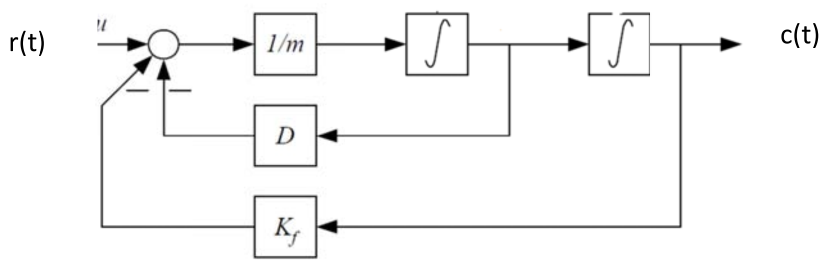
Solution

System 1: $G_1(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2+1}$

System 2: $G_2(s) = \frac{Z(s)}{Y(s)} = \frac{s}{s-6}$

Overall System Transfer function: $G(s) = \frac{Z(s)}{X(s)} = \frac{s}{(s^2+1)(s-6)}$

Problem 4 (6 pts)
Figure below represents the block diagram of an LTI control system.



Determine the Differential equation relating the output c(t) to the input r(t).

Solution

System Transfer function: $\frac{C(s)}{R(s)} = \frac{\frac{1}{ms^2}}{1+\frac{D}{ms}+\frac{K_f}{ms^2}} = \frac{1}{ms^2+Ds+K_f}$

$$m\frac{d^2c(t)}{dt^2} + D\frac{dc(t)}{dt} + K_f c(t) = r(t)$$

<u>Problem 5 (6 pts)</u>

The forward transfer of a unity feedback control system is given by

$$G(s) = \frac{(s-7)}{(s+2)(s^2 - s + 6)}$$

Use the Routh-Hurwitz stability criterion to determine the location of the poles of the system in each half of the complex plane.

$$\frac{\text{Output}}{\text{Input}} = \frac{(s-7)}{(s+2)(s^2 - s + 6) + (s-7)} = \frac{s-7}{s^3 + s^2 + 5s + 5}$$ **2 pts**

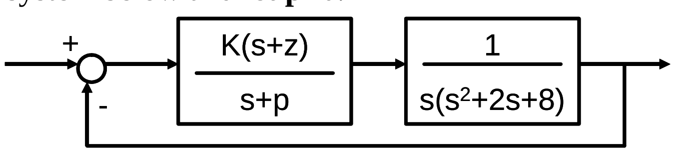
s^3	1	5
s^2	1	5
s^1	0	
s^1	2	
s^3	5	

$$A(s) = s^2 + 5 \Rightarrow \frac{dA(s)}{ds} = 2s. \quad A(s) = 0 \Rightarrow s = \pm j\sqrt{5}$$

Pole locations (1, 2, 0)

<u>Problem 6 (6 pts)</u>

Consider the system below and **let p=0**.



Determine conditions on K and z so that the system is stable. (3 pts)

<u>Answer</u>

Closed-loop TF: $TF = \dfrac{K(s+z)}{s^4 + 2s^3 + 8s^2 + Ks + Kz}$

RH Table

s^4	1	8	Kz
s^3	2	K	
s^2	$\dfrac{16-K}{2}$	Kz	
s^1	$\dfrac{K(16-4z-K)}{16-K}$		
s^0	Kz		

1. 0<K<16
2. 0<z<4

Determine all possible conditions on K and z so that the system will be marginally stable. (3 pts)
Answer
The open-loop TF A(s) is given by

$$A(s) = \frac{K(s+z)}{s^2(s^2+2s+8)}$$

The intersection with the imaginary axis takes place when

$$1 + \mathrm{A(j}\omega) = 0 \Rightarrow \omega^4 - j2\omega^3 - 8\omega^2 + jK\omega + Kz = 0$$

$$2\omega^3 - K\omega = 0 \qquad (1)$$

$$\omega^4 - 8\omega^2 + Kz = 0\,(2)$$

In (1), either $\omega = 0$ or $K = 2\omega^2$

When $\omega = 0$, Kz=0. When $K = 2\omega^2$, (2) implies that $\frac{K^2}{4} - 4K + Kz = 0 \Rightarrow \mathrm{K(K - 16 + 4z) = 0}$

The above implies

$$\mathrm{K + 4z = 16}$$

Problem 7 (5 pts)
Find the feasible pole locations of the second order system whose transfer function is given by

$$\frac{\mathrm{Y(s)}}{\mathrm{R(s)}} = \frac{\omega_{\mathrm{n}}^2}{\mathrm{s}^2 + 2\varsigma\omega_n s + \omega_n^2}$$

Given that the settling time $\mathrm{t_s} = 5\% \le 3\,\mathrm{sec}$ and the percent overshoot $\mathrm{M_p} \ge 4.3\%$.

Solution

$$\mathrm{M_p} = e^{-\pi\varsigma/\sqrt{1-\varsigma^2}} = 0.043 \Rightarrow \varsigma \approx \frac{\sqrt{2}}{2}$$

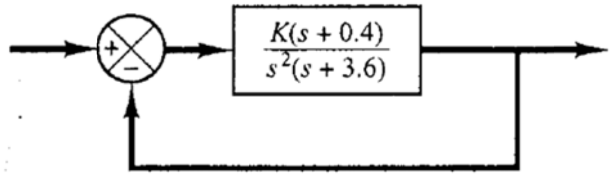
$$\mathrm{T_s} = \frac{3}{\varsigma\omega_{\mathrm{n}}} = 3 \Rightarrow \omega_n \approx \sqrt{2}$$

$$\frac{\mathrm{Y(s)}}{\mathrm{R(s)}} = \frac{2}{\mathrm{s}^2 + 2s + 2}$$

The poles are located at s=-1-j and s=-1+j

Problem 8 (7 pts)

A unity feedback control system, shown below, has a variable gain K.



It is intended to plot the root locus of this system. You are asked to determine

1. Number of braches that goes to infiity if any. (1 pt)
   2
2. Origin and angles of the asysmototes if any. (1 pt)
   OA=-1.6
   AA=90 degrees and -90 degrees

3. Part of the real axis that is a part of root locus if any. (1 pt)
   [-3.6, -0.4]

4. Breakinreal axis points or breakout from real axis points if any. (1 pt)
   Break in at: -1.3

5. Intersection with the imaginary axis if any. (1 pt)
   None

6. Plot the root locus in the below space. (2 pts)